

Heating and Cooling of Hot Accretion Flows by Non Local Radiation

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ABSTRACT

We consider non-local effects which arise when radiation emitted at one radius of an accretion disk either heats or cools gas at other radii through Compton scattering. We discuss three situations:

1. Radiation from the inner regions of an advection-dominated flow Compton cooling gas at intermediate radii and Compton heating gas at large radii.
2. Soft radiation from an outer thin accretion disk Compton cooling a hot one- or two-temperature flow on the inside.
3. Soft radiation from an inner thin accretion disk Compton cooling hot gas in a surrounding one-temperature flow.

We describe how previous results are modified by these non-local interactions. We find that Compton heating or cooling of the gas by the radiation emitted in the inner regions of a hot flow is not important. Likewise, Compton cooling by the soft photons from an outer thin disk is negligible when the transition from a cold to a hot flow occurs at a radius greater than some minimum $R_{\text{tr,min}}$. However, if the hot flow terminates at $R < R_{\text{tr,min}}$, non-local cooling is so strong that the hot gas is cooled to a thin disk configuration in a runaway process. In the case of a thin disk surrounded by a hot one-temperature flow, we find that Compton cooling by soft radiation dominates over local cooling in the hot gas for $\dot{M} \gtrsim 10^{-3} \alpha \dot{M}_{\text{Edd}}$, and $R \lesssim 10^4 R_{\text{Schw}}$. As a result, the maximum accretion rate for which an advection-dominated one-temperature solution exists, decreases by a factor of ~ 10 , compared to the value computed under an assumption of local energy balance.

Subject headings: accretion, accretion disks – radiation mechanisms: nonthermal – radiative transfer

1. Introduction

Hot optically thin accretion flow solutions are often used to model X-ray binaries and active galactic nuclei. In addition to the original two-temperature model of Shapiro, Lightman & Eardley (1976) and the corona model of Haardt & Maraschi (1991), two new classes of advection-dominated

models have been proposed recently: a two-temperature solution (Narayan & Yi 1995, Abramowicz et al. 1995), and a one-temperature solution (Esin et al. 1996).

Advection-dominated models differ from the usual accretion solutions in that only a fraction of the viscously dissipated energy is radiated locally in the disk. The remainder of the generated energy is stored in the gas as entropy, resulting in a very hot, optically thin, quasispherical flow. If the accreting object is a black hole, as it is believed to be the case for several X-ray binaries and all active galactic nuclei, the stored energy is carried by the gas inside the horizon, so that the luminosity of the system can be orders of magnitude smaller than is predicted by standard accretion theory. The gas in advection-dominated solutions cools primarily through bremsstrahlung, synchrotron and inverse Compton scattering processes. Since the accreting electrons are heated to temperatures of up to 10^{10} K, the spectra of such systems span ~ 10 orders of magnitude in energy, from synchrotron radio emission to $\sim 400\text{keV}$ X-rays produced via Compton cooling of hot electrons. Because two-temperature advection-dominated solutions have low radiative efficiency and hard power-law spectra, they have been successful in reproducing the observed properties of various low-luminosity X-ray sources including Sagittarius A* (Narayan, Yi, & Mahadevan 1995), the black hole nova A0620-00 in quiescence (Narayan, McClintock & Yi 1996), and the central source in NGC 4258 (Lasota et al. 1996). They also appear to be relevant to luminous systems (Narayan 1996). It is not yet clear if one-temperature solutions are relevant to any observed system.

Though detailed numerical spectra of advection-dominated flows include global radiative transfer effects (Narayan, McClintock & Yi 1996; Narayan 1996; Narayan, Barret & McClintock 1997), analytical calculations of the physical properties of these accretion solutions, which we briefly review in §2, generally ignore the effects of non-local radiative transfer (Narayan & Yi 1995; Esin et al. 1996). This is a dangerous approximation. The optical depth for electron scattering in such flows is generally much lower than unity, and therefore, radiation emitted at one radius can potentially change the energy balance of the gas at quite different radii. It is the purpose of this paper to investigate the importance of this effect. We analyze three different but related issues.

First, in the advection-dominated models the bulk of the luminosity is emitted from the innermost region of the disk ($R < 100R_{Schw}$), where most of the gravitational energy is dissipated and the cooling efficiency of the gas is higher. The radiative flux from this region can have two effects on the rest of the flow, as we discuss in §3.1. The hot radiation can Compton heat (§3.1.1) the outer regions ($R > 1000R_{Schw}$), where the gas is cooler (Shvartsman 1971, Ostriker et al. 1976, Grindlay 1978). On the other hand, cold but numerous synchrotron photons can contribute to cooling of the gas (§3.1.2) closer in ($100R_{Schw} < R < 1000R_{Schw}$).

Next, the hot solutions based on the one-temperature advection model (Esin et al. 1996) do not extend down to the last stable orbit for $\dot{M} > 0.003\alpha^2\dot{M}_{Edd}$. At accretion rates higher than this limit the only possible configuration is a standard thin disk (Shakura & Sunyaev 1973; Frank, King & Raine 1992), surrounded by a hot flow. Since most of the energy is dissipated in the thin

disk, the number density of the cold photons emitted in the inner regions can be much greater than the number density of photons emitted locally by the hot gas, and we expect that the hot flow will suffer significant extra cooling via Compton scattering of the thin disk photons. This issue is discussed in §3.2.

Finally, both one- and two-temperature advection-dominated solutions can be surrounded by a thin accretion disk, with the boundary between them at some radius R_{tr} , where the value of R_{tr} depends on the parameters of the model. Here again the cold photons emitted by the thin disk can Compton cool electrons in the nearby hot flow. We analyze this situation in §3.3.

Just as the radiation from the thin disk affects the energy balance of the hot flow, the cold gas itself is affected by irradiation from the advection-dominated zone. In §3.4 we investigate whether this process affects our conclusions from §3.2 and §3.3. We summarize and discuss the results in §4.

2. Advection-Dominated Accretion Flows

In this section we review the basic equations, derived by Narayan & Yi (1995), that describe local properties of advection-dominated accretion flows in terms of five basic parameters: mass of the accreting object, M , mass accretion rate, \dot{M} , radial distance from the accreting object, R , standard viscosity coefficient, α (Shakura & Sunyaev 1973), and the ratio of gas pressure to total pressure, β .

2.1. Self-Similar Solution

The total pressure in a vertically averaged, axisymmetric accretion flow is given by

$$p = \rho c_s^2, \quad (2-1)$$

where ρ is the gas density and c_s is the local isothermal sound speed. We assume that the gas is in equipartition with a tangled magnetic field, so the total pressure is the sum of the gas and magnetic pressures, $p = p_g + p_m$. In the ionized gas, the expression for the gas pressure is

$$p_g = \beta \rho c_s^2 = \frac{\rho k T_i}{\mu_i m_u} + \frac{\rho k T_e}{\mu_e m_u}, \quad (2-2)$$

where T_i , T_e , and μ_i , μ_e are temperatures and molecular weights of ions and electrons respectively. The magnetic pressure due to the isotropic tangled magnetic field is given by

$$p_m = (1 - \beta) \rho c_s^2 = \frac{B^2}{24\pi}. \quad (2-3)$$

Note that this equation differs from the one adopted by Narayan & Yi (1995) by a factor of 1/3.

Narayan & Yi (1995) have derived a self-similar solution for advection-dominated accretion flows that obey mass, energy, and momentum conservation. Combining this solution with the equation for the gas pressure, and scaling mass in units of the solar mass, $M = mM_\odot$, accretion rate in Eddington units, $\dot{M} = \dot{m}\dot{M}_{Edd} = \dot{m} 1.39 \times 10^{18} m \text{ g s}^{-1}$, and radii in Schwarzschild units, $R = rR_{Schw} = r 2.95 \times 10^5 m \text{ cm}$, we obtain

$$n_e = 1.33 \times 10^{19} \frac{\dot{m}}{m\alpha(c_3 r)^{3/2}} \text{ cm}^{-3}, \quad (2-4)$$

$$q_v^+ = 1.84 \times 10^{21} \frac{\epsilon' \dot{m} \sqrt{c_3}}{m^2 r^4} \text{ erg cm}^{-3} \text{ s}^{-1}, \quad (2-5)$$

$$T_i + 1.08 T_e = 6.66 \times 10^{12} \frac{\beta c_3}{r} \text{ K}, \quad (2-6)$$

where n_e is the electron density, q_v^+ is the rate of viscous energy dissipation per unit volume, and we have taken $\mu_i = 1.23$ and $\mu_e = 1.14$, as required for a gas composition of 75% H and 25% He. The parameters c_3 and ϵ' are defined as

$$c_3 = \frac{2(5 + 2\epsilon')}{9\alpha^2} \left\{ \left[1 + \frac{18\alpha^2}{(5 + 2\epsilon')^2} \right]^{1/2} - 1 \right\} \quad (2-7)$$

and

$$\epsilon' = \frac{1}{f} \left(\frac{5/3 - \gamma}{\gamma - 1} \right) = \frac{1 - \beta}{f}; \quad (2-8)$$

where f is the fraction of viscously dissipated energy that is stored in the gas, and the expression for the adiabatic exponent γ is derived in Appendix A.

2.2. Heating and Cooling Processes and Local Energy Balance

2.2.1. Heating of Electrons

Since ions are considerably more massive than electrons, viscous heating affects primarily the ions (Shapiro, Lightman, & Eardley 1976; Phinney 1981; Rees et al. 1982). However, cooling of the hot plasma occurs mainly through electrons, so the energy has to be transferred in some way from ions to electrons. In their analysis, Narayan & Yi (1995) (following Shapiro, Lightman, & Eardley [1976]) assumed that the energy transfer can occur only through Coulomb interactions at a volume transfer rate, q_{ie} , the expression for which was derived by Stepney & Guilbert 1983. Since $q_v^+ \propto \dot{m} \propto n_i$ (Eq. [2-5]) and $q_{ie} \propto n_i n_e$ (Stepney & Guilbert 1983), at low densities, Coulomb

interactions are less efficient than viscous energy dissipation, giving rise to two-temperature accretion flows with ions much hotter than electrons.

In addition to Coulomb scatterings, some non-thermal coupling mechanisms have been proposed recently (e.g. Begelman & Chiueh 1988). If such mechanisms are more efficient than Coulomb coupling, they can restore the thermal equilibrium between the ions and electrons, creating a one-temperature accretion flow (Esin et al. 1996). In that case, the question of energy transfer from ions to electrons is settled by simply setting $T_i = T_e$ at all times.

2.2.2. Local Cooling Processes

The fraction $(1 - f)$ of the energy generated by viscous heating must be radiated by the gas. Cooling of the hot plasma occurs via three processes: bremsstrahlung, synchrotron radiation, and inverse Compton scattering of low energy bremsstrahlung and synchrotron photons:

$$q^- = q_{br}^- + q_{sync}^- + q_C^-. \quad (2-9)$$

Bremsstrahlung and Line Emission The free-free emission in a plasma is produced in electron-electron and electron-ion interactions. The corresponding cooling rates were derived by Stepney & Guilbert (1983) and Svensson (1982). The total bremsstrahlung cooling rate per unit volume is the sum of the two contributions, $q_{br}^- = q_{ee}^- + q_{ei}^-$.

At large radii, the gas is cold enough to allow line cooling, which becomes more efficient than free-free emission below 10^8 K. At these temperatures electron-electron bremsstrahlung emission is negligible, and we use a cooling function that includes electron-ion bremsstrahlung and line cooling (updated version; original version was described by Raymond, Cox, & Smith [1976]).

Synchrotron Emission In a hot plasma with equipartition magnetic field, synchrotron radiation is an important cooling mechanism. An expression for the spectrum of optically thin synchrotron emission from a relativistic Maxwellian distribution of electrons was derived by Mahadevan, Narayan, & Yi (1996). However, below some critical frequency, ν_c , the emission becomes self-absorbed and the optically thin emission formalism no longer applies. To estimate ν_c we equate the synchrotron emission from a sphere of radius R to the Rayleigh-Jeans blackbody emission from the surface of that sphere. Below ν_c the emission is optically thick and we use the blackbody estimate, while above ν_c we use the expression derived by Mahadevan et al. (1996). Integration over frequency yields the total cooling rate per unit volume (Esin et al. 1996).

Inverse Compton Scattering Both bremsstrahlung and synchrotron emission mechanisms produce mainly soft photons, which are then upscattered by the hot electrons in the accreting gas.

The optical depth to electron scattering is simply $\tau_e = 2n_e\sigma_T H$, where H is the scale height of the flow determined through the conditions for vertical hydrostatic equilibrium. After each scattering, the energy of a photon is increased by a factor $A = 1 + 4\theta_e + 16\theta_e^2$ (Rybicki & Lightman 1979), where $\theta_e = kT_e/m_e c^2$ is the dimensionless electron temperature. The average ratio of the photon energy at escape to its initial energy is given by (Esin et al. 1996)

$$\eta = e^{(A-1)s}[1 - P(j_m + 1)] + \eta_{max}P(j_m + 1, s), \quad (2-10)$$

where $P(a, x) = [1/\Gamma(a)] \int_0^x t^{a-1} e^{-t} dt$ is the incomplete gamma function and

$$s = \tau_e + \tau_e^2, \quad (2-11)$$

$$j_m = \ln(\eta_{max})/\ln(A), \quad (2-12)$$

$$\eta_{max} = 3kT_e/h\nu. \quad (2-13)$$

The synchrotron spectrum is strongly peaked at ν_c , so we can estimate the Compton cooling rate due to the scattering of synchrotron photons as $q_{C,s}^- = (\eta(\nu_c) - 1)q_{sync}^-$. On the other hand, the spectrum of bremsstrahlung radiation is practically flat between the synchrotron self-absorption frequency ν_c (we found that the bremsstrahlung self-absorption frequency is generally lower than ν_c for the range of accretion flow parameters that is relevant for this work) and exponential cut-off at kT/h . Below ν_c free-free emission falls off as Rayleigh-Jeans blackbody and Comptonization is not important, since the photons are more likely to be absorbed than scattered. Consequently, we can estimate the emission per unit frequency as $q_{br,\nu}^- \simeq q_{br}^- / \left(\frac{kT}{h} - \nu_c\right)$. Then the Compton cooling rate is given by the integral $q_{C,br}^- = \int_{\nu_c}^{\frac{kT}{h}} q_{br,\nu}^- [\eta(\nu) - 1] d\nu$.

The total Compton cooling rate is simply $q_C^- = q_{C,s}^- + q_{C,br}^-$.

2.2.3. Energy Balance

The local energy balance in the gas requires that a fraction $(1 - f)$ of the viscously dissipated energy is radiated away. This gives us the condition

$$(1 - f)q_v^+ = q^-, \quad (2-14)$$

which together with the equation of state and self-similar solution described in §2.1 is sufficient to obtain a closed set, necessary to solve for all one-temperature flow parameters. For a two-temperature solution, we need another equation to determine T_i and T_e separately. We obtain it by requiring that the net cooling rate has to be equal to the heating rate of the electrons:

$$q^- = q_{ie}. \quad (2-15)$$

2.3. Physical Properties of One- and Two-Temperature Flows

The set of equations described in §2.1-2.2 allows all the major parameters describing the accreting gas to be solved for self-consistently for given values of M , \dot{M} , R , α , and β . The resulting flows are quasispherical, with $H \sim R$, optically thin, and quite hot. Their properties were described in detail by Narayan & Yi (1995) and Esin et al. (1996), and here we give a brief summary.

As was shown first by Rees et al. (1982) advection-dominated accretion solutions are limited to values of \dot{m} that lie below a maximum mass accretion rate, $\dot{m}_{crit}(r)$. Above this limiting value, the radiative efficiency of the gas is so high due to the increased plasma density, that the flow cools down to the standard thin accretion disk configuration. Figure 1(a) shows a comparison of the critical accretion rate as a function of R computed for one- and two-temperature models. At large radii, $r > 10^3$, the two calculations give identical results, with $\dot{m}_{crit} \propto r^{-3/2}$. Note that previous papers by Narayan & Yi (1995) and Abramowicz et al. (1995) have $\dot{m}_{crit} \propto r^{-1/2}$ in the outer regions. Our much stronger dependence of the critical accretion rate on radius is due to including line cooling, which is significantly more efficient than bremsstrahlung at low temperatures. Closer to the accreting object, \dot{m}_{crit} decreases steeply as $\dot{m}_{crit} \propto r^{3/2}$ in a one-temperature model, but is roughly independent of r in a two-temperature model (the slight decrease in \dot{m}_{crit} seen on the figure is most likely an artifact of the self-similar assumption we used to derive the solution; when proper boundary conditions are used at the inner edge of the disk, this decrease is no longer present [c.f. Narayan 1996]).

To understand why the two solutions behave in this way, we show in Figure 1(b) the radial temperature profile of the accreting gas for a $10M_\odot$ black hole, with $\dot{M} = 10^{-5}\dot{M}_{Edd}$, $\alpha = 0.1$ and $\beta = 0.5$. At large radii, the ion and electron temperature in a two-temperature flow are equal to each other and to the gas temperature in a one-temperature flow. In that region, the Coulomb energy transfer from ions to electrons is efficient and the two solutions are identical. However, in the region inside $1000R_{Schw}$, the electron temperature in a two-temperature solution remains roughly constant, whereas the ions stay at near virial temperatures with $T_i \propto r^{-1}$. The gas in a one-temperature flow, on the other hand, necessarily finds an equilibrium temperature with $T_e = T_i = T_0$. The temperature T_0 obviously lies between T_e and T_i of the two-temperature solution. At $\sim 10^{10}$ K, the rate of synchrotron and Compton cooling of the gas is very sensitive to the temperature of the electrons, and since electrons in one-temperature solutions are hotter than electrons in two-temperature solutions, the former has a much higher radiative efficiency. This in turn leads to lower \dot{m}_{crit} .

The difference in temperatures between the two solutions also leads to different values of the advection parameter f . Figures 1(c) and 1(d) show contours of constant f as a function of radius and accretion rate, for two- and one-temperature models respectively. Two-temperature flows are advection-dominated, $f \sim 1$ everywhere, except for a narrow region near the \dot{M}_{crit} boundary (shown as a heavy line). On the other hand, the inner regions of single-temperature accretion disks

are cooling-dominated, $f \lesssim 0.5$, for $\dot{m} \gtrsim 10^{-5} \alpha^2$, because they have higher electron temperatures. In fact, one-temperature hot accretion flows at high \dot{M} have radiative efficiency close to 15%, which means they radiate away nearly all the gravitational energy lost in the accretion process (Esin et al. 1995). By comparison, the efficiency of two-temperature accretion solutions can be orders of magnitude less.

3. Effects of Nonlocal Radiation

The formalism described in the previous section ignores the effects of non-local radiative transfer. It assumes that the photons emitted at one radius interact only with the ‘local’ gas, i.e. gas within a region of unit radial width in $\log(r)$. We can check the validity of this assumption by estimating the optical depth for electron scattering in the radial direction predicted by advection models. The optical depth from radius r_{min} to r is simply $\tau_r(r_{min}, r) = \int_{r_{min}}^r n_e \sigma_T R_{Schw} dr$. After substituting the expression for n_e given by Eq. (2-4), where we take $c_3 \simeq 0.3$ (see Appendix B), and integrating, we obtain

$$\tau_r(r_{min}, r) = \tau_r(r_{min}) \simeq 1.8 \left(\frac{\dot{m}}{10^{-2}} \right) \left(\frac{0.1}{\alpha} \right) \left(\frac{3}{r_{min}} \right)^{1/2}, \quad (3-1)$$

assuming that $r \gg r_{min}$. Thus, two-temperature accretion disks are optically thin if $\dot{M} \lesssim 3 \times 10^{-2} \alpha \dot{M}_{Edd}$, which means that the photons can travel freely through the disk and be scattered by electrons far from where they were emitted, heating or cooling the gas as a result of scattering.

One-temperature accretion disks are geometrically thinner in the inner region than two-temperature disks, because they are able to cool more efficiently (Esin et al. 1996). As a result, both n_e and τ_r are higher. However, even one-temperature flows are optically transparent below $\dot{M} \sim 3 \times 10^{-3} \alpha \dot{M}_{Edd}$.

In the remainder of this section we attempt to quantify how Compton scattering of non-local radiation will affect the flow parameters calculated based on local analysis. In each case we estimate analytically the region in the parameter space where non-local effects can be important, then we compute numerically how the properties of the solutions are affected in this region.

3.1. Photons from the Hot Inner Region

Both one- and two-temperature accretion flows produce most of their emission from within $100 R_{Schw}$, where most of the gravitational potential energy is released. The inner region also

has the highest electron temperatures so that the emerging photons are hotter than the gas in the surrounding flow. When this radiation interacts with the gas in the outer regions of the disk, two processes can take place. Where the gas is relatively hot, less energetic photons will cool the electrons through inverse Compton scattering. On the other hand, where the temperature of the electrons is less than the effective temperature of the radiation, the electrons will be heated via Compton scattering.

The amount of the heating or cooling can be easily estimated if we assume that each photon undergoes at most one scattering at any given radius (a reasonable assumption, since the Compton y -parameter (Rybicki & Lightman 1979) of the accreting gas at $R > 100R_{Schw}$ is always less than unity). With this approximation, the rate of energy transfer from photons to electrons per unit volume, via Compton scattering, is just

$$q_{nl} = \sigma_T c n_e e^{-\tau_r} \int n_{\gamma,\nu} \Delta E_\nu d\nu \text{ erg s}^{-1} \text{ cm}^{-3}, \quad (3-2)$$

where $n_{\gamma,\nu}$ is the number density of the incident photons per unit frequency and $\Delta E_\nu = E_{\gamma,\text{in}} - E_{\gamma,\text{fin}}$ is the energy lost by the photon in each scattering. When q_{nl} is positive, the electrons are heated by the incident photon flux, when q_{nl} is negative, the electrons are cooled. We approximate $n_{\gamma,\nu}$ as $n_{\gamma,\nu} = F_\nu / ch\nu \simeq L_\nu / 4\pi R^2 ch\nu$, where L_ν is the luminosity per unit frequency emerging from the region within $100R_{Schw}$. ΔE_ν is given by Rybicki & Lightman (1979)

$$\Delta E_\nu = E_{\gamma,\text{in}} - E_{\gamma,\text{fin}} \simeq \frac{h\nu}{m_e c^2} (h\nu - 4kT_e). \quad (3-3)$$

Since ΔE_ν can be at most as large as the original photon energy, Eq. (3-3) is valid only for nonrelativistic photons with $h\nu < m_e c^2$. To extend this formula to higher photon energies, we have to replace $h\nu / m_e c^2$ by $h\nu / (m_e c^2 - h\nu)$, to ensure that $\Delta E_\nu < h\nu$. However, numerical calculations of q_{nl} (described in §3.1.1 and §3.1.2) show that this modification has a very small effect on the final result and so for simplicity we use Eq. (3-3) in our analytic estimates.

3.1.1. Heating of the Gas

Let us consider first the rate of heating of the gas by the energetic photons. From Eq. (3-3) it is clear that only photons with energies $h\nu > 4kT_e(r)$ can contribute to the heating, whereas photons with lower energies will cool the gas. The high energy cutoff of the spectrum is determined by the electron temperature in the inner regions, $T_{e,\text{in}}$, since inverse Compton scattering cannot produce photons with energies higher than $\sim 3kT_{e,\text{in}}$ (Rybicki & Lightman 1979). Therefore, to obtain an upper limit on the amount of heating, the integral in Eq. (3-2) has to be evaluated from $\nu = 4kT_e(r)/h$ to $\nu = 3kT_{e,\text{in}}/h$. Clearly then, significant amount of Compton heating can occur only if $T_e(r) \ll T_{e,\text{in}}$, which restricts us to the region outside $1000R_{Schw}$ (see Figure 1[b]).

The precise shape of L_ν depends sensitively on the value of the accretion rate. However, a reasonable upper limit to the effect of heating can be obtained by assuming that the spectrum is flat, i.e. equal energy is emitted per logarithmic interval in frequency, and that the luminosity emitted from the inner region is approximately equal to the total luminosity of the disk. Since the spectra generally span ~ 10 orders of magnitude in frequency (e.g. Narayan, Yi, & Mahadevan 1995; Lasota et al. 1996), we can write $L_\nu \simeq L/(10 \log_e 10 \nu)$, with L being the total emitted luminosity. Using this approximation, and substituting Eqs. (2-4) and (3-3) into Eq. (3-2), we obtain an approximate expression for the rate of heating caused by non-local radiation from smaller radii:

$$q_{nl}^+ \simeq 1.1 \times 10^{-18} \frac{\dot{m} e^{-\tau_r} L}{m^3 \alpha r^{7/2}} \left(\frac{T_{e,in}}{10^9 \text{ K}} \right) \text{ erg s}^{-1} \text{ cm}^{-3}. \quad (3-4)$$

where we assumed $c_3 \simeq 0.3$ (Appendix B) and used the fact that $T_{e,in} \gg T_e$ to simplify the final result.

Compton heating of electrons by hot photons becomes important when it is comparable to the rate of energy transfer from ions to electrons, so to find the region of the parameter space where non-local heating must be taken into account, we impose the condition $q_{nl}^+/q_{ie} = q_{nl}^+/q^- \geq 1$. The expression for q^- is derived in Appendix B, and the left-hand side is given by Eq. (3-4).

For two-temperature accretion disks, this condition requires that

$$\dot{m} \gtrsim 2.3 \times 10^{-2} \left(\frac{\alpha}{0.1} \right) \left(\frac{1000}{r} \right)^{1/2} \left(\frac{10^9 \text{ K}}{T_{e,in}} \right) e^{\tau_r}, \quad (3-5)$$

for $r > 1000$. The values of \dot{m} and r for which this relation is satisfied lie above the dashed line on Figure 2(a), computed assuming $\alpha = 0.3$, $\beta = 0.5$ and $\tau_r \ll 1$. One can see that the region where significant Compton heating is expected to occur is small.

In reality, since we used upper limits in estimating q_{nl}^+ , our analysis overestimates the magnitude of this effect. In order to find the exact values, we have solved the complete set of equations discussed in §2 numerically, and computed the values of electron density, plasma temperatures, and magnetic field strength in a two-temperature accretion flow for various values of r and \dot{m} . Using these parameters we calculate the local cooling rate, q^- , and the shape of the spectrum of radiation emerging from the inner region (see Narayan, Barret & McClintock [1997] for details). Then the integral in Eq. (3-2) can be computed numerically using exact values for n_e , $L_\nu e^{-\tau_r}$, and ΔE , to obtain q_{nl}^+ . On Figure 2(a) we have plotted the contours of constant q_{nl}^+/q^- as dotted lines. It is clear from this figure that non-local radiation mainly Compton *cools* the accreting gas, since the values of this ratio are for the most part negative. Where Compton heating does occur, it makes up at most a few percent of the energy transferred to electrons from ions, and can therefore be completely neglected.

For one-temperature flows, the total disk luminosity can be estimated from Esin et al. (1996)

$$L_{1-t} \simeq \begin{cases} 1.3 \times 10^{38} \dot{m} \text{ erg s}^{-1} & \text{for } \dot{m} \gtrsim 10^{-5} \alpha^2, \\ 1.3 \times 10^{43} \left(\frac{\dot{m}^2}{\alpha^2} \right) \left(\frac{0.5}{\beta} \right) \text{ erg s}^{-1} & \text{for } \dot{m} \lesssim 10^{-5} \alpha^2. \end{cases} \quad (3-6)$$

Then the condition $q_{nl}^+ \geq q^-$ requires that

$$\dot{m} \gtrsim 4.6 \times 10^{-5} \left(\frac{\alpha}{0.1} \right) \left(\frac{1-\beta}{0.5} \right) \left(\frac{10^3}{r} \right)^{1/2} \left(\frac{10^{10} \text{ K}}{T_{e,in}} \right) e^{\tau_r}, \quad (3-7)$$

for $r > 1000$. The region where Eq. (3-7) is valid lies above the dashed line on Figure 2(b), for $\alpha = 0.1$, $\beta = 0.5$, and $\tau_r \ll 1$. Note that the area affected by non-local heating is larger than the similar estimate for a two-temperature flow on Figure 2(a). This happens because one-temperature disks are more luminous than their two-temperature counterparts. However, this result is again an overestimate. As we did in the case of a two-temperature flow, we computed the ratio q_{nl}/q^- numerically, solving the flow equations appropriate for a one-temperature plasma. The constant value contours of this ratio are shown on Figure 2(b) (dotted lines). The results show that Compton heating is indeed stronger here than it was in a two-temperature gas. It is most important for values of the mass accretion rate $\dot{m} \sim \dot{m}_{crit}(r=3)$, since at higher \dot{m} the hot flow can not extend all the way to $3R_{Schw}$, and at lower \dot{m} the optical depth is low and Comptonization is inefficient, so that fewer high energy photons are produced, capable of heating the gas in the outer flow. However even at its maximum, non-local Compton heating does not exceed $\sim 10\%$ of the local heating of electrons via viscous dissipation.

We conclude that Compton heating can be neglected within the advection-dominated zone both for two- and one-temperature flows.

3.1.2. Cooling of the Gas

The photons with energies $h\nu < 4kT_e$ cool the gas through inverse Compton scattering. An upper limit on the amount of cooling induced by non local radiation can be obtained by assuming that all photons are emitted at $\nu \ll 4kT_e/h$, so that Eq. (3-3) reduces to $\Delta E \simeq -h\nu(4\theta_e + 16\theta_e^2) = h\nu(1 - A)$, where the term $16\theta_e^2$ is added to account for relativistic effects at $\theta_e \gtrsim 1$. Since at $r > 100$ the electron temperature in both one- and two-temperature flows varies as $T_e \simeq 10^{12}\beta \text{ K}/r$ (see Figure 1(a) and Eq. [B1]), the quantity $A - 1$ can be written as $A - 1 \simeq 670\beta/r + (670\beta/r)^2$. With this simplification, the integral in Eq. (3-2) is trivial and the expression for the cooling rate evaluates to

$$q_{nl}^- = 5.0 \times 10^{-17} \left[\frac{670\beta}{r} + \left(\frac{670\beta}{r} \right)^2 \right] \frac{\dot{m} e^{-\tau_r} L}{m^3 \alpha r^{7/2}} \text{ erg s}^{-1} \text{ cm}^{-3}, \quad (3-8)$$

where we have set $q_{nl}^- = |q_{nl}|$.

Substituting Eq. (B5) for L and imposing the requirement $q_{nl}^-/q^- \geq 1$, we find that in two-temperature flows photons from the inner region will contribute to the cooling of the gas at

$r > 100$ when

$$\dot{m} \gtrsim 6 \times 10^{-4} \left(\frac{\alpha}{0.1} \right) \left(\frac{r}{100} \right)^{1/2} \left(\frac{0.5}{\beta} \right) \left[1 + 3.4 \left(\frac{100}{r} \right) \left(\frac{\beta}{0.5} \right) \right]^{-1} e^{\tau_r}. \quad (3-9)$$

For $\alpha = 0.3$, $\beta = 0.5$, and $\tau_r \ll 1$, this inequality defines a region that lies above the dashed curve on Figure 2(c). We see that the extra cooling can be important only for high values of the accretion rate, when the electron density in the flow is large enough to scatter the outgoing photons from smaller radii.

The results of an exact numerical calculation are also shown on Figure 2(c). We have computed q_{nl}^- and q^- as described in §3.1.1, and plotted the curve with $q_{nl}^-/q^- = 1$ as a solid line. The shaded region inside this curve, where non-local cooling can affect the accretion flow, is very small. To investigate how the flow properties change in the affected region, we have included q_{nl}^- into our numerical calculations as an extra cooling mechanism, replacing Eq. (2-9) by $q^- = q_{br}^- + q_{sync}^- + q_C^- + q_{nl}^-$. As a result, the equilibrium electron temperature of the accreting gas and the advection parameter f decrease slightly, to compensate for the increased cooling. However, since most of the emission produced by the flow comes from within $100R_{Schw}$, there is practically no change in the observed spectrum.

To estimate where non-local cooling is significant in one-temperature flows, we substitute Eq. (3-6) into Eq. (3-8) and compare the result to the local cooling rate, q^- , obtaining the following condition on \dot{m} and r

$$\dot{m} \gtrsim 1.2 \times 10^{-7} \left(\frac{\alpha}{0.1} \right) \left(\frac{1-\beta}{\beta} \right) \left(\frac{r}{100} \right)^{1/2} \left[1 + 3.4 \left(\frac{100}{r} \right) \left(\frac{\beta}{0.5} \right) \right]^{-1} e^{\tau_r}, \quad (3-10)$$

for $r > 100$. The region of the parameter space where this inequality is satisfied lies above the dashed line on Figure 2(d), computed for $\alpha = 0.1$, $\beta = 0.5$, and $\tau_r \ll 1$. The results of a detailed numerical computation are shown as a shaded region on the same figure.

From comparison of Figures 2(c) and 2(d), it is clear that non-local cooling is much more important in one-temperature than in two-temperature accretion flows. However, in both cases, the only effect is to decrease the gas temperature and the advection parameter in the affected regions of the flow, by a modest amount. However, these regions do not produce most of the observed emission. Therefore, we conclude that the radiation emitted in the inner regions of a hot flow does not have significant observable effect on the gas in the outer regions.

3.2. Cold Photons from the Inner Thin Disk

In a one-temperature accretion flow model, the value of the critical accretion rate, \dot{m}_{crit} , increases as $\propto r^{3/2}$ for $r \lesssim 1000$ (see Fig. 1[a]). For $\dot{m} > \dot{m}_{crit}$, the only equilibrium solution

is the thin accretion disk. As a result, when the value of the accretion rate is in the range $10^{-3}\alpha^{3/2} \lesssim \dot{m} \lesssim \alpha^{3/2}$, the only possible flow configuration is a thin disk surrounded by a hot quasi-spherical flow. Cold photons emitted by the thin disk will Compton cool the hot gas around it. This is a similar problem to the one we discussed in §3.1.2. However, since the luminosity considered here is higher because of the higher efficiency of the thin disk, we expect that non-local Compton cooling will be more important.

In this scenario, non-local cooling is no longer limited to the outer regions of the disk, where the accreting gas has low density and temperature and therefore, multiple scatterings of the incident photons are not important. In fact, when we compute the Compton y -parameter we find that it can be much greater than unity for $r \ll 100$. Thus, to estimate the rate of non-local Compton cooling, we need to modifying the expression for ΔE_ν to include the effect of multiple scatterings:

$$\Delta E_\nu = E_{\gamma,\text{in}} - E_{\gamma,\text{fin}} = h\nu(1 - \eta(\nu)A), \quad (3-11)$$

with η given by Eq. (2-10). In general, the value of η depends on the frequency of the scattered radiation, but for simplicity we set it equal to some average value $\tilde{\eta}$. As in §3.1.2, we assume that the radiation is cold and the factor $h\nu$ is small as compared to kT_e . With that approximation, the Compton cooling rate becomes

$$q_{nl}^- = \sigma_T n_e e^{-\tau_r} F_c (\tilde{\eta}A - 1). \quad (3-12)$$

where $F_c = L_c/4\pi R^2$ is the cold photon flux and L_c is the total luminosity of the thin disk. Since the standard thin disk has approximately Keplerian energetics, we can estimate L_c as the energy loss between the outer and inner radii of the disk:

$$L_c \simeq \frac{GM\dot{M}}{2R_{Schw}} \left(\frac{1}{3} - \frac{1}{r_{out}} \right). \quad (3-13)$$

Combining Eqs. (3-12) and (3-13), we obtain the following expression for the cooling rate

$$q_{nl}^- \simeq 1.5 \times 10^{23} \left(\frac{\dot{m}^2}{m^2 \alpha} \right) \left(\frac{e^{-\tau_r}}{r^{7/2}} \right) (\tilde{\eta}A - 1) \left(\frac{1}{3} - \frac{1}{r_{out}} \right). \quad (3-14)$$

Compton cooling becomes important when it exceeds local cooling of the gas. Comparing Eqs. (3-14) and (B6) we find that q_{nl}^- is greater than q^- at all radii, provided that $\dot{m} \gtrsim 10^{-3}\alpha^2$ (necessary for the thin disk to be present) and $\tilde{\eta}A - 1 > 1$. Thus, if the thin disk is present, it will always Compton cool the hot flow around it.

To obtain more detailed results, we computed the quantities q^- and q_{nl}^- numerically. In our calculation, we assumed that all the radiation from the thin disk is emitted at the frequency corresponding to a blackbody of temperature $T_{BB} \simeq (3GM\dot{M}/8\pi R^3 \sigma_B)^{1/4}$ (Frank, King, & Raine 1992) evaluated at $R = 3R_{Schw}$, and computed the corresponding $\tilde{\eta}$ at each radius. The region where $q_{nl}^-/q^- \geq 1$ is shaded on Figure 3(a). The flow in this region will be cooled significantly by photons from the thin disk. Including non-local Compton cooling directly in our calculations

shows that because of the increase in the cooling rate, the gas near the inner edge of the hot flow collapses to a thin disk, and the critical accretion rate line itself moves downwards. This means that the thin disk extends slightly further outward and produces more cold radiation. After a few iterations, this process converges; the new \dot{M}_{crit} is shown on Figure 3(a) as a dashed line. Due to extra cooling, the maximum accretion rate for which hot one-temperature solution exists, decreased by a factor of ~ 10 . For $\dot{M} < \dot{M}_{crit}$, where hot solutions are still allowed, the equilibrium temperature of the accreting gas decreases (Figure 3[b]), so that the observed spectrum contains less high energy flux.

3.3. Cold Photons from the Outer Thin Disk

Outside $1000R_{Schw}$, the critical accretion rate for both one- and two-temperature solutions decreases with radius as $\dot{m}_{crit} \propto r^{-3/2}$. Therefore, for any value of \dot{m} there is a critical radius r_{crit} beyond which the only possible flow configuration is the thin disk. Therefore, we expect to have accretion flows where the gas forms a cool thin disk for $r > r_{crit}$, but switches to a hot advection-dominated state for $r < r_{crit}$. Many of the models published in the literature (e.g. Narayan et al. 1996) are of this form.

In such configurations, the photons produced in the cold outer flow may be expected to Compton cool the hot gas inside r_{crit} . The cooling rate per unit volume is given by Eq. (3-12), the only unknown being the cold photon flux, F_c .

In general, F_c is a function of both radius, R , and the height above the plane of the thin disk, $z = R \sin \theta$. Taking into account the full geometry of the problem, we write $F_c(R, z)$ as a double integral:

$$F_c(R, z) = \int_{R_{tr}}^{\infty} dR' \, 2 \int_0^{\pi} R' d\phi \frac{\sigma_B T_c^4 \sin \psi}{\pi x^2}, \quad (3-15)$$

where R_{tr} is the transition radius between the hot and cold parts of the accretion disk, $T_c(R')$ is the surface temperature of the thin disk at radial position R' , x is the distance from the emitting element in the thin disk to the scattering element in the hot flow, ϕ is the azimuthal angle, and $\sin \psi = z/x$. Since we are working with a height-averaged set of equations, we replace $F_c(R, z)$ with its average over a spherical shell of radius R :

$$\tilde{F}_c(R) = \int_0^{\pi/2} \cos \theta d\theta \int_{R_{tr}}^{\infty} dR' \, 2 \int_0^{\pi} R' d\phi \frac{\sigma_B T_c^4 \sin \psi}{\pi x^2} \quad (3-16)$$

The emission per unit area of the thin disk can be approximated as $\sigma_B T_c^4 \simeq 3GM\dot{M}/8\pi R^3$ (Frank, King, & Raine 1992), and substituting expressions for x and ψ in terms of R , R' and θ we obtain

$$\tilde{F}_c(R) = \frac{3GM\dot{M}R}{4\pi^2} \int_{R_{tr}}^{\infty} \frac{dR'}{R'^2} \int_0^{\pi/2} d\theta \cos \theta \sin \theta \int_0^{\pi} \frac{d\phi}{(R'^2 + R^2 - 2R'R \cos \theta \cos \phi)^{3/2}} \quad (3-17)$$

This integral has to be evaluated numerically, but a reasonable estimate can be obtained by replacing $\sin \psi$ by $\sin \tilde{\psi}$, where $\tilde{\psi}$ is some effective angle averaged over the entire spherical shell. Since $\sin \psi = R \sin \theta / (R'^2 + R^2 - 2R'R \cos \theta \cos \phi)^{1/2}$, setting $\phi = 0$, $\theta = \cos^{-1} R/R_{tr}$, and $R' = R_{tr}$ gives an upper limit of $\sin \psi = R/R_{tr}$. Then the three integrals can be evaluated analytically:

$$\tilde{F}_c(R) = \frac{3GM\dot{M}}{4\pi} \frac{R}{R_{tr}} \left\{ \frac{1}{2R_{tr}R^2} + \frac{R^2 - R_{tr}^2}{4R_{tr}^2R^3} \left[\ln \left(1 + \frac{R}{R_{tr}} \right) - \ln \left(1 - \frac{R}{R_{tr}} \right) \right] \right\}. \quad (3-18)$$

In the limit when $R \sim R_{tr}$, Eq. (3-18) reduces to $\tilde{F}_c \simeq 3GM\dot{M}/8\pi R_{tr}^2 R$; when $R \ll R_{tr}$, $\tilde{F}_c \simeq GM\dot{M}R/4\pi R_{tr}^4$, so that when $R/R_{tr} \rightarrow 0$, the cold photon flux becomes completely negligible.

Clearly, Compton cooling will be strongest near the boundary between the hot and cold flows, so an upper limit on the amount of extra cooling can be obtained by replacing Eq. (3-18) by the limiting expression for \tilde{F}_c when $R \sim R_{tr}$. Substituting this result into Eq. (3-12) and imposing the familiar constraint $q_{nl}^-/q^- \geq 1$, we find that external cooling is significant in the hot flow near the boundary if

$$r_{tr} \lesssim 200 \left(\frac{r}{r_{tr}} \right)^3 \left(\frac{\alpha}{0.1} \right)^2 \left(\frac{0.5}{\beta} \right)^2 \left(\frac{\tilde{\eta}A(r) - 1}{2} \right)^2, \quad (3-19)$$

for $\dot{m} \gtrsim 10^{-3}\alpha^2$, and

$$r_{tr} \lesssim 10^3 \left(\frac{r}{r_{tr}} \right)^3 \left(\frac{\dot{m}}{10^{-5}} \right)^2 \left(\frac{0.5}{1 - \beta} \right)^2 \left(\frac{0.1}{\alpha} \right)^2 (\tilde{\eta}A(r) - 1)^2, \quad (3-20)$$

for $\dot{m} \lesssim 10^{-3}\alpha^2$. Since the critical outer radius for a hot flow is always greater than $10^3 R_{Schw}$ (see Figure 1[b]), this result shows that the thin disk radiation has no effect on the hot gas, so long as $R_{tr} \sim R_{crit}$.

The cooling from the outer disk can, however, be significant if $R_{tr} \ll R_{crit}$. There is no well-developed theory at this time for the transition radius R_{tr} and one could in principle imagine a situation where the gas remains in a cold state down to a much smaller radius than R_{crit} . If we consider such a model, the extra cooling of the hot gas at $R \sim R_{tr}$ can be quite important. To show this, we computed the ratio q_{nl}^-/q^- numerically, using the full expression for \tilde{F}_c (as given by Eq. [3-18]). The results for one- and two-temperature hot flows are shown on Figures 4(a) and 4(b) respectively, where we have plotted q_{nl}^-/q^- evaluated at $R = R_{tr}$ for different values of \dot{m} and r_{tr} . We can see that external cooling begins to dominate when $r_{tr} \sim 10^2 - 10^3$, in reasonable agreement with our analytic approximation.

As the transition radius decreases further, the ratio of non-local to local cooling at the boundary between a two-temperature hot flow and a thin disk becomes more extreme, increasing up to ~ 200 for high accretion rates, and causing a significant decrease in the equilibrium temperature of the hot gas. Note however, that only a relatively thin layer near the boundary is affected by extra cooling. Away from the boundary, the cold photon flux decreases linearly with r

and the ratio of non-local to local cooling falls steeply with decreasing radius (see Eqs. [3-12] and [B6]):

$$\frac{q_{nl}^-}{q^-} \propto \frac{r \cdot r^{-3/2}}{r^{-4}} = r^{7/2}. \quad (3-21)$$

Because of this strong dependence on r , the net effect on the hot flow is negligible, as long as the hot gas in the boundary layer is in *stable thermal equilibrium*. However, the hot equilibrium configuration cannot be maintained for an arbitrarily strong cold photon flux. At some point, the total cooling in the boundary layer becomes so strong, that the gas temperature is forced significantly below the virial value, i.e. the hot quasispherical flow collapses into a cold thin disk. When this occurs, the transition radius effectively moves inwards, which means that the hot gas in the new boundary layer will experience even stronger external cooling, and will also inevitably collapse. In this fashion, a runaway instability develops that causes the entire hot flow to cool down to the standard Shakura & Sunyaev disk.

We have computed numerically the values of the transition radius at which this instability occurs for different mass accretion rates. This was done by incorporating q_{nl}^- directly into the calculations and decreasing r_{tr} until the hot stable accretion flow solution at $r \sim r_{tr}$ disappeared. The resulting critical values of r_{tr} are shown on Figure 5 as dashed lines. We see that this instability is an important consideration only for large values of the mass accretion rate. For $\dot{M} \lesssim 0.1 \times \alpha^2 \dot{M}_{Edd}$, the thin disk has virtually no effect on the hot flow, irrespectively of where the transition between the cold and hot phases occurs.

In a single-temperature hot flow, the value of the ratio q_{nl}^-/q^- evaluated at $r = r_{tr}$ never becomes much higher than unity (see Figure 4[b]), even for small values of the transition radius. The reason for this is that at $r < 100$, one-temperature flows are cooling-dominated, i.e. local cooling is very efficient. At $r < r_{tr}$ the cooling ratio behaves roughly in accordance with Eq. (3-21), so that external cooling is completely unimportant there.

3.4. Irradiation of the Thin Disk by the Hot Flow

In calculating the energy flux from the thin disk in §3.2 and §3.3 we used the standard (Frank, King, & Raine 1992) formalism and ignored the effects of irradiation of the cold gas by the advection flow. In reality, most of the hot photons incident on the thin disk are reprocessed there, so that the net emission from the thin disk is a blackbody at a higher temperature than predicted by the standard theory. To test whether irradiation is important, we need to compute the ratio of the incident energy flux to the viscous dissipation rate per unit area of the thin disk.

Using cylindrical geometry for the hot flow, the incident energy per unit area of an inner thin

disk can be written as a triple integral

$$Q_{inc}(R') = \int_{R_{tr}}^{\infty} dR \, 2 \int_0^{\pi} R d\phi \int_0^{H(R)} dz \frac{q^-(R) \sin \psi}{4\pi x^2}, \quad (3-22)$$

where R , R' , z , x , and ϕ are defined as in §3.3. Since the thin disk is surrounded with a one-temperature accretion flow, for which f is significantly below unity, we can set $q^- = q_v^+$, where q_v^+ is given by Eq. (B3). The result can be expressed in terms of elliptical integrals, but to get a rough estimate of $Q_{inc}(R')$ we assume that $R' \ll R$, i.e. we restrict our analysis to radii well inside the transition radius. Then to zeroth order in (R'/R) the ratio of the incident to the viscously dissipated energy evaluates to

$$\frac{Q_{inc}(R')}{\sigma_B T_c(R')} = 0.01 \left(\frac{r}{r_{tr}} \right)^3. \quad (3-23)$$

Thus, irradiation contributes at most a few percent to the energy output of the thin disk. This result was confirmed by integrating Eq. (3-22) numerically, with the correct expression for q^- in the case of a one-temperature flow. We conclude that irradiation is entirely negligible in this scenario, because the inner thin disk subtends a small solid angle from the point of view of the hot flow, and most of the energy is dissipated in the thin disk. By comparison, in the disk and corona model (e.g. Haardt & Maraschi 1991), where irradiation is known to play an important role, the hot corona lies directly on top of the cold gas, so that roughly half of all photons emitted in the corona are absorbed by the disk.

The expression for the energy incident on an outer thin disk differs from Eq. (3-22) only in the limits of the outermost integral; in this case we integrate R from the last stable orbit, $3R_{Schw}$, to R_{tr} . Again, we restrict ourselves to regions away from the transition radius, i.e. $R' \gg R$, and assume that $q^- = q_v^+$. Then to first order in R/R' , the ratio of the incident to viscously dissipated energy per unit area is independent of R' :

$$\frac{Q_{inc}(R')}{\sigma_B T_c(R')} = 0.15 \ln \left(\frac{r_{tr}}{3} \right). \quad (3-24)$$

Numerical calculations show that even at $R' \sim R$, the result increases by at most $\sim 20\%$.

This calculation shows that irradiation of the outer thin disk becomes important only at large transition radii, where, according to our results from §3.3 the presence of the thin disk does not affect the hot flow. Note also that large transition radii are allowed only for low accretion rates, when the flow becomes strongly-advection dominated, and the result in Eq. (3-24) decreases by an additional factor of $(1 - f)$. On the other hand, if the transition occurs within $10^3 R_{Schw}$, irradiation will increase the flux from the thin disk by at most a factor of 2. This has only a small effect on our calculations of the critical transition radius, and qualitatively our results remain the same.

4. Discussion

In this paper we have explored how non-local radiation transfer affects the properties of hot one- and two-temperature advection-dominated solutions described by Esin et al. (1996), Narayan & Yi (1995) and Abramowicz et al. (1995). Since these solutions are optically thin, radiation emitted at one radius could in principle induce significant cooling or heating of the accreting gas at some other radius, through Compton scattering. We considered three different sources of non-local radiation: a luminous inner region of a hot disk ($R < 100R_{Schw}$), a thin disk inside a hot one-temperature gas, and a thin Shakura & Sunyaev disk surrounding a hot advection-dominated flow.

We found that two-temperature solutions are not affected by the radiation from their hot interior. These solutions are strongly advection-dominated everywhere except near the outer boundary of the disk, R_{crit} , (see Fig. 1[c]), and the inner part simply does not produce enough hot photons to induce significant heating of the outer layers. On the other hand, the gas density of the flow at $R > 100R_{Schw}$ is so small, that with electron temperatures of $T_e \lesssim 10^9$ K, Compton cooling by non-local radiation can not play an important role either.

The inner parts of one-temperature flows are cooling-dominated and produce more radiation than their two-temperature counterparts. However, single-temperature solutions are limited to lower values of the mass accretion rate (see Figure 1[a]), and since hot photons are produced mainly by Comptonization, which is not an efficient process at low \dot{M} , no heating of the outer flows occurs. On the other hand, we find that Compton cooling by the photons from the interior dominates over local cooling processes for $10^2R_{Schw} \lesssim R \lesssim 10^4R_{Schw}$ and $\dot{M} \gtrsim 10^{-7}\dot{M}_{Edd}$. However, since the bulk of the observable radiation comes from inside $100R_{Schw}$, the changes in the flow properties caused by extra cooling do not appreciably affect the overall spectrum of the system.

Compton cooling by the radiation from the inner thin disk does have a significant impact on one-temperature flows, since it occurs at higher \dot{M} , when Compton scattering is very effective. The extra cooling induced by the thin disk photons is so strong, that the maximum accretion rate for which the one-temperature model has an equilibrium solution decreases by a factor of ~ 10 (Figure 3[a]).

Finally, we found that the radiation from an outer thin disk does not have any effect on a hot flow that extends beyond $\sim 10^3R_{Schw}$. At this distance, the gas is too cold for significant Compton cooling to take place. When the transition radius between the hot and cold gas moves closer to the accreting object, the non-local cooling of two-temperature gas near the boundary can become very large. However, this effect is limited to the layers of the hot flow very close to the transition between the hot and cold gas; away from this boundary, the ratio of non-local to local cooling decreases as $r^{7/2}$. Therefore, external cooling does not affect the regions of the hot gas where most of the observable radiation is produced and can be disregarded until it is so strong that it forces the hot flow near the boundary to cool to a thin disk configuration. When this happens, the transition radius decreases and the hot gas in the new boundary layer experiences even stronger

external cooling, which causes it to collapse in its turn. The resulting runaway process continues until the entire flow assumes the thin disk configuration. This phenomenon can be important if the hot advection-dominated flows are formed by evaporation of the thin disk near the accreting black hole (Narayan & Yi 1995). In that case, the maximum accretion rate at which such hot flow can form is in fact less than \dot{M}_{crit} computed based on the local assumption.

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A. Modified Derivation of the Energy Equation And Adiabatic Exponent for the Accreting Gas

One of the conditions that the physical parameters of the accretion flow must satisfy is the conservation of energy. Following Abramowicz et al. (1988) and Narayan & Yi (1994) we can write this condition as

$$\rho v T \frac{ds}{dR} = f q^+, \quad (\text{A1})$$

where s is the entropy per unit mass of the gas and other quantities are defined in §2.1.

To derive the expression for ds/dR we model the gas in the accreting flow as a combination of an ideal monatomic gas (a justified assumption since the accretion flow consists mainly of ionized hydrogen) and tangled magnetic fields. Radiation pressure was shown to be unimportant in one-temperature flows (Esin et al. 1996) and since two-temperature solutions are even less luminous, the radiation pressure can safely be ignored. Then the total pressure in the gas is the sum of the thermal ideal gas pressure, and the pressure due to the magnetic field (Eqs. [2-2] and [2-3]) and can be written as

$$p = p_g + p_m = \frac{\rho k T}{\mu m_u} + \frac{B^2}{24\pi} = \frac{\rho k T}{\beta \mu m_u}, \quad (\text{A2})$$

where we have defined $T = T_i$, $\mu = \mu_i \mu_e / (\mu_e + \mu_i T_e / T_i)$ and $\beta = p_g / p$. In general, μ is a function of both T_i and T_e ; however, for simplicity we restrict ourselves to the two limiting cases when $T_i \gg T_e$ and $\mu = \mu_i$, or $T_i = T_e$ and $\mu = \mu_i \mu_e / (\mu_e + \mu_i)$.

The internal energy of the gas is the sum of the kinetic energy of the particles (since the gas is assumed to be monatomic) and the energy stored in the magnetic field. The total internal energy per unit mass is then

$$u = \frac{3}{2} \frac{k T}{\mu m_u} + \frac{1}{\rho} \frac{B^2}{8\pi} = \frac{3}{2} \frac{k T}{\mu m_u} + \frac{3(1-\beta)}{\beta} \frac{k T}{\mu m_u} = \frac{(6-3\beta)}{2\beta} \frac{k T}{\mu m_u} \quad (\text{A3})$$

In a quasistatic process, the first law of thermodynamics requires that

$$T ds = du + p dV = \left(\frac{\partial u}{\partial T} \right)_V dT + \left(\frac{\partial u}{\partial V} \right)_T dV + p dV, \quad (\text{A4})$$

where $V = 1/\rho$ is the volume per unit mass. But u is a function of T only, which means that the first term vanishes, $(\partial u / \partial V)_T = 0$. The second term we evaluate as

$$\left(\frac{\partial u}{\partial T} \right)_V = \frac{du}{dT} = \frac{(6-3\beta)}{2\beta} \frac{k}{\mu m_u}. \quad (\text{A5})$$

Finally, we divide both sides of Eq. (A4) by T and obtain the following relation for the entropy

$$ds = \frac{(6-3\beta)}{2\beta} \frac{k}{\mu m_u} \frac{dT}{T} + \frac{\rho k}{\beta \mu m_u} d \left(\frac{1}{\rho} \right). \quad (\text{A6})$$

For an adiabatic process we set $ds = 0$. Following Clayton (1983, Eq. [2-121c]) we define the corresponding adiabatic exponent γ as

$$\gamma = \Gamma_3 = \frac{\left(\frac{1}{\beta} \frac{k}{\mu m_u}\right)}{\left(\frac{(6-3\beta)}{2\beta} \frac{k}{\mu m_u}\right)} + 1 = \frac{8-3\beta}{6-3\beta} \quad (\text{A7})$$

With this definition, we integrate Eq. (A6) to obtain an expression for the entropy of the gas up to a constant factor:

$$s = \frac{k}{\beta \mu m_u} \frac{1}{\gamma - 1} \ln(c_s^2 \rho^{1-\gamma}) + \text{const.} \quad (\text{A8})$$

Now we are in a position to write down the final form for the energy conservation equation. Evaluating the derivative of s with respect to R and substituting it into Eq. (A1), yields the same expression as given by Narayan & Yi (1994)

$$\frac{\rho v}{\beta(\gamma - 1)} \frac{dc_s^2}{dR} - v c_s^2 \frac{d\rho}{dR} = f q^+, \quad (\text{A9})$$

but with a different value for γ , namely Eq. (A7).

B. Approximate Analytic Expression for the Viscous Heating and Local Cooling

We need to obtain simplified expressions for the rate of viscous energy dissipation and local cooling of the accreting gas, as functions of the global parameters, $m, \dot{m}, \alpha, \beta$ and r only.

B.1. Two-Temperature Accretion Disks

In two-temperature flows the ion temperature is always nearly virial and scales as $M/R \propto 1/r$. The electron and ion temperatures are equal when $r \gtrsim 1000$, but T_e stays nearly constant at $\sim 10^9$ K in the inner region of the disk. We find that the sum $T_i + T_e$ is well approximated by

$$T_i + T_e \simeq \frac{10^{12} \text{ K}}{r} \left(\frac{\beta}{0.5} \right), \quad (\text{B1})$$

for all values of \dot{m} , α , and β . Substituting this expression into Eq. (2-6) gives us $c_3 \simeq 0.3$. Inverting Eq. (2-7), we obtain an expression for ϵ' in terms of c_3 :

$$\epsilon' = \frac{1}{2} \left(\frac{9\alpha^2}{x} - \frac{x}{2} - 5 \right), \quad x = \frac{9c_3\alpha^2}{2}. \quad (\text{B2})$$

The second term in brackets can be neglected as long as $\alpha \lesssim 0.5$. Then Eq. (B2) simplifies to $\epsilon' \simeq 0.8$.

Substituting these results into Eq. (2-5), we find that the total energy per unit volume dissipated in the disk can be written as

$$q_v^+ \simeq 8.1 \times 10^{20} \frac{\dot{m}}{m^2 r^4} \text{ erg s}^{-1} \text{ cm}^{-3}. \quad (\text{B3})$$

As required by the energy balance equation, the rate of local cooling is given by $q^- = (1-f)q_v^+$. In general, f is a function of r as well as \dot{m} , α , and β , but for simplicity we use the radially averaged value which can be easily estimated by integrating q^- over the volume of the disk and comparing the result with the total emitted luminosity, L_{2-t} :

$$L_{2-t} \simeq (1-f) \int_{r_{min}}^{\infty} q_v^+ 2H 2\pi R dR. \quad (\text{B4})$$

We adopt the value for L_{tot} estimated by Mahadevan (1997)

$$L_{2-t} \simeq \begin{cases} 1.3 \times 10^{38} \left(\frac{m\dot{m}^2}{\alpha^2} \right) \left(\frac{\beta}{0.5} \right) \text{ erg s}^{-1}, & \dot{m} \gtrsim 10^{-3} \alpha^2, \\ 2.5 \times 10^{34} m\dot{m} \left(\frac{1-\beta}{0.5} \right) \text{ erg s}^{-1}, & \dot{m} \lesssim 10^{-3} \alpha^2; \end{cases} \quad (\text{B5})$$

Combining Eqs. (B3), (B4), and (B5) and setting $r_{min} = 3$, we obtain the following expression for the local cooling

$$\begin{aligned}
 q^- &= (1-f)q_v^+ = 9.3 \times 10^{-18} \frac{L_{2-t}}{m^3 r^4} \\
 &= \begin{cases} 1.2 \times 10^{21} \frac{\dot{m}^2}{m^2 \alpha^2 r^4} \left(\frac{\beta}{0.5} \right) \text{ erg s}^{-1} \text{ cm}^{-3}, & \text{for } \dot{m} \gtrsim 10^{-3} \alpha^2, \\ 2.3 \times 10^{17} \frac{\dot{m}}{m^2 r^4} \left(\frac{1-\beta}{0.5} \right), & \text{for } \dot{m} \lesssim 10^{-3} \alpha^2. \end{cases} \quad (\text{B6})
 \end{aligned}$$

B.2. One-Temperature Accretion Disks

Two-temperature flows become effectively single-temperature when $r > 1000$, since there $T_e = T_i$ (Esin et al. 1996, Fig. 1[b]). Moreover, in the region $30 \lesssim r \lesssim 1000$ the gas temperature, T , in one-temperature disks is nearly as high as T_i in equivalent two-temperature solutions, and the values of the advection parameter f are similar as well. Therefore, in the outer regions Eqs. (B3) and (B6) apply equally well to both models.

Inside $\sim 30 R_{Schw}$, as one-temperature disks become thinner, denser and cooling-dominated, both q_v^+ and q^- increase faster with decreasing radius than the corresponding quantities in two-temperature disks. In this regime, we can use Eqs. (B3) and (B6) as lower limits.

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Figure Captions

Figure 1. (a) Critical mass accretion rate, \dot{M}_{crit} , as a function of radius for one-temperature (dashed line) and two-temperature (solid line) models, plotted for two different values of the viscosity parameter α . Hot advection-dominated solutions exist below the respective lines; above the lines the only equilibrium configuration available to the accretion flow is a standard thin disk. (b) Radial temperature profile for a flow with a fixed accretion rate of $10^{-5}M_{Edd}$. (c) Contours of constant advection parameter f for a two-temperature accretion flow. The critical accretion rate is shown as a heavy line. (d) Contours of constant advection parameter f for a one-temperature accretion flow. The critical accretion rate is shown as a heavy line.

Figure 2. Critical mass accretion rate (heavy line), \dot{M}_{crit} , plotted in Eddington units, as a function of radius for two-temperature (a, c) and one-temperature (b, d) models. (a) and (b) Above the dashed line lies the region where significant Compton heating by the photons emitted at $R < 100R_{Schw}$ is expected to occur, based on an analytic approximation (Eqs. [3-5] and [3-7]). The dotted lines are the contours of constant q_{nl}/q^- computed numerically, labeled by the values of this ratio. Positive numbers correspond to a net heating, negative numbers indicate that non-local radiation produces a net cooling. Note that the ratio of non-local to local heating is always significantly below unity. (c) and (d) Shaded area corresponds to the region of the accretion flow where local cooling processes are less efficient than Compton cooling by the photons emitted at $R < 100R_{Schw}$, computed numerically. The dashed lines are the contours of $q_{nl}^-/q^- = 1$, estimated analytically (Eqs. [3-9] and [3-10]).

Figure 3. (a) Critical mass accretion rate, \dot{M}_{crit} , plotted in Eddington units (solid heavy line), for a one-temperature model with a $10M_{\odot}$ black hole, viscosity parameter $\alpha = 0.1$, and ratio of gas pressure to total pressure $\beta = 0.5$. For accretion rate values above \dot{M}_{crit} the only equilibrium configuration available to the accretion flow is a standard thin disk. The region of the hot accretion flow for which non-local cooling due to scattering of the cold photons emitted by the thin disk is more efficient than local cooling processes is shaded. When the non-local cooling is included in the calculations, the critical accretion rate decreases. The new \dot{M}_{crit} curve is plotted as a dashed heavy line. (b) Compares the temperature profile of the flow with (thin line) and without (heavy line) extra cooling due to the inner thin disk.

Figure 4. The ratio of non-local cooling rate due to scattering of the radiation emitted in the outer thin disk, to the local cooling rate in the gas near the transition layer between the hot and cold flows, plotted as a function of the transition radius for two-temperature (a) and one-temperature (b) models. Different curves are labeled by the values of the accretion rate in Eddington units. Note that for $R_{tr} \lesssim 100R_{Schw}$, this ratio is significantly smaller in one-temperature models which have much stronger local cooling than their two-temperature counterparts.

Figure 5. Critical mass accretion rate, \dot{M}_{crit} , plotted in Eddington units (solid lines) for two-temperature models with viscosity parameter $\alpha = 0.1$ and 0.3 . The dashed lines correspond to the minimum allowed values of the transition radius between the hot flow and the outer thin disk. If the transition occurs in the region to the left of the dashed line, Compton cooling of the hot gas by the thin disk photons is so strong that the entire flow settles to the thin disk configuration.











